Much of mathematics is about discovering robust kinds of structure which organize and illuminate large areas of the subject. Perhaps the most basic organizing concept of our thought is space. It leads us to the homotopy category, which captures many of our geometric intuitions but also arises unexpectedly in contexts far from ordinary spaces. Still more is this true of the 'stable homotopy' category, which sits midway between geometry and algebra.

The theme of my lectures is the strangeness and the ubiquity of the homotopy and stable homotopy categories, and how they give us new ideas of what a space is, and why manifolds and spaces with algebraic structure play such a special role. The first lecture will be a non-technical survey of things discussed in more detail in the following two, such as the dual ways of thinking of a space in terms of the paths in it and in terms of its algebra of functions; also, what abstract homotopy theory is, and how it leads us to new geometrical ideas and new analogues of the stable homotopy category.

One of the biggest steps in disengaging homotopy from spaces was Kan's theory of simplicial objects; a subtheme is the different ways in which this can be viewed and the different directions in which it can lead.

The second lecture focuses on the relation between spaces and their algebras of functions, algebraic K-theory, and the nature of noncommutative spaces and 'quantization' — what it means for a noncommutative object to be a deformation of a normal space. We see how the stable homotopy category sits inside a noncommutative stable category. I shall speculate on the place of Floer homotopy theory in this picture.

The third lecture is partly about how manifolds relate to stable homotopy, and the role of Poincaré duality and quadratic forms. It is also about the 'higher homotopies' which lie below the surface of the naive homotopy category. This is another angle on the interpolation between commutativity and noncommutativity, more from the perspective of traditional algebraic topology.