Models for the Leaf Space of a Foliation

Ieke Moerdijk

The aim of this talk is to explain and compare some approaches to the leaf space (or "transverse structure") of a foliation. A foliation is a certain partition $\mathcal{F}$ of a manifold M into immersed submanifolds, the leaves of the foliation. Identifying each of the leaves to a single point yields a very uninformative, "coarse" quotient space, and the problem is to define a more refined quotient $M/\mathcal{F}$, which captures aspects of that part of the geometric structure of the foliation which is constant and/or trivial along the leaves.

It is possible to distinguish (at least) three approaches to this problem. One is in the spirit of non-commutative geometry [4], and uses the duality between the manifold M and the ring $C_c^\infty(M)$ of compactly supported smooth functions on M. The *quotient* $M/\mathcal{F}$ is then modelled, dually, by an extension of this ring $C_c^\infty(M)$, the so-called convolution algebra of the foliation. Completion of such convolution algebras leads one into C^*-algebras. Important invariants are the cyclic type (i.e. Hochschild, cyclic, periodic cyclic) homologies and the K-theory of these convolution and C^*-algebras.

A second approach, which predates non-commutative geometry, is to construct a quotient "up to homotopy". Like all such homotopy colimits in algebraic topology, this construction takes the form of a classifying space. This approach goes back to Haefliger, who constructed a classifying space $B\Gamma_q$ for foliations of codimension q, as the leaf space of the "universal" foliation [9, 2]. Important invariants are the cohomology groups of these classifying spaces, in particular the universal or characteristic classes coming from the cohomology of the universal leaf space $B\Gamma_q$.

A third approach, even older, is due to Grothendieck. Not surprisingly, Grothendieck uses the 'duality' between the space M and the collection of all its sheaves, which form a topos $\mathrm{Sh}(M)$. The quotient $M/\mathcal{F}$ can then be constructed as a suitable topos "$\mathrm{Sh}(M/\mathcal{F})$", consisting of sheaves on M which are invariant along the leaves in a suitable sense. One can then apply the whole machinery of [17], and study the Grothendieck fundamental group of $\mathrm{Sh}(M/\mathcal{F})$, its sheaf cohomology groups, etc. etc..

Central to all these approaches is the construction [18] of a smooth groupoid out of the foliated manifold $(M, \mathcal{F})$, called the holonomy groupoid and denoted $\mathrm{Hol}(M, \mathcal{F})$. The three approaches above then become special instances of the general procedure of associating to a smooth (or "Lie") groupoid G a convolution algebra $C_c^\infty(G)$, a classifying space BG, or a classifying topos $\mathrm{Sh}(G)$. Of these, the last one is intuitively closest to a manifold. For example, there are immediate natural constructions of the differential forms on such a topos, of its tangent bundle (another topos mapping to $\mathrm{Sh}(G)$), and so on for almost any construction of differential topology and geometry I can think of. This is partly caused by the fact that the Lie groupoids arising in this context are all (equivalent to) étale groupoids. The diagram below provides a schematic summary of the situation. In this lecture, I will first give more precise definitions and references for the notions occurring in this diagram and then explain some relations between the three legs.

Foliation $(M, \mathcal{F})$

↓

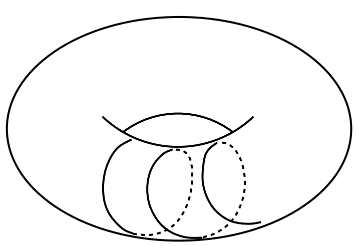
Lie groupoid G

↙ ↓ ↘

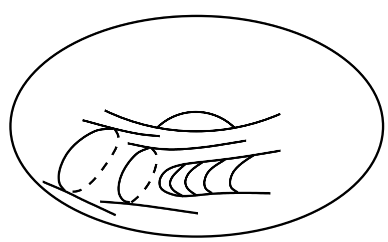
Convolution algebra $C_c^\infty(G)$ (non-commutative geometry)	Classifying space BG or $B\,\mathrm{Emb}(G)$ (algebraic topology)	Classifying topos $\mathrm{Sh}(G)$ (topos theory)

1. Foliations

Let M be a manifold of dimension n. A foliation $\mathcal{F}$ of M is an integrable subbundle $\mathcal{F} \subseteq TM$ of the tangent bundle. Integrability means that if two vector fields on M belong to $\mathcal{F}$ then so does their Lie bracket. If $\mathcal{F}$ is of rank p, the foliation is said to be of dimension p and of codimension $q = n - p$. Integrability implies that through each point $x \in M$ there is a unique connected p-dimensional immersed submanifold L_x which is everywhere tangent to $\mathcal{F}$, called the leaf of $\mathcal{F}$ through x. These leaves form a partition of M. This partition is locally trivial in the sense that at each point x there is a chart $\varphi\colon \mathbb{R}^n \to U$ (where U is a neighborhood of x) such that for $\mathbb{R}^n = \mathbb{R}^p \times \mathbb{R}^q$ the plaques $\varphi(\mathbb{R}^p \times \{t\})$ are exactly the connected components of the intersections of U with the leaves. (A specific leaf may pass through U in different plaques.) Here are two easy and well-known examples of foliations.



Kronecker foliation.



Reeb foliation.

The Kronecker foliation $\mathcal{K}$ of the torus $T = S^1 \times S^1$ is the foliation given by the 1-dimensional subbundle of vectors in $\mathbb{R}^2$ with a fixed irrational slope. The leaves are immersed copies of the real line which wrap around the torus infinitely often, each leaf being dense. The Reeb foliation $\mathcal{R}$ of the solid torus has one compact boundary leaf, and its other leaves are planes. If you imagine the solid torus as obtained from the solid cylinder $\mathbb{R} \times D$ by identifying (t, x) and $(t + 1, x)$ for every point x on the disk D, then the interior of $\mathbb{R} \times D$ is foliated by planes which are stacked upon each other as infinitely deep salad bowls, and the Reeb foliation is the quotient. To get rid of the boundary, one can construct S^3 as the union of two solid tori. Then the union of Reeb foliations is the Reeb foliation of S^3, with one compact leaf.

The theory of foliations is a vast subject, for which there are many good introductions, e.g. the books by Camacho and Neto, Godbillon, Hector and Hirsch, Tondeur, and others.

2. Lie Groupoids

A groupoid G is a small category all of whose arrows are isomorphisms. It thus has a set G_0 of objects $x, y, \ldots$ and a set G_1 of arrows $g, h, \ldots$. Each arrow has a source $x = s(g)$ and a target $y = t(g)$, written $g\colon x \to y$. Two arrows g and h with $s(g) = t(h)$ can be composed as $gh\colon s(h) \to t(g)$. This composition is associative, has a unit $1_x\colon x \to x$ at each object x, and has a two-sided inverse $g^{-1}\colon t(g) \to s(g)$ for each arrow g. All the structure is contained in a diagram

$$G_2 \xrightarrow{m} G_1 \xrightarrow{i} G_1 \underset{t}{\overset{s}{\rightrightarrows}} G_0 \quad (\text{with } u\colon G_0 \to G_1) \tag{1}$$

(s = source, t = target, i = inverse, u = units, m = composition, defined on $G_2 = \{(g, h) \in G_1 \times G_1 : s(g) = t(h)\}$. The groupoid G is said to be *smooth* or *Lie* if G_0 and G_1 are smooth manifolds, each of the structure maps in (1) is smooth, and s, t are submersions so that G_2 is a smooth manifold as well. The classical reference for Lie groupoids is [11].

For a Lie groupoid G and a point $x \in G_0$, the arrows $g\colon x \to x$ form a Lie group G_x, called the *isotropy group* at x. A Lie groupoid G is called a *foliation groupoid* if each of its isotropy groups is discrete. All the groupoids arising from foliations have this property.

There is an obvious notion of *smooth functor* or *homomorphism* $\varphi\colon H \to G$ between Lie groupoids. It consists of two smooth maps (both) denoted $\varphi\colon H_0 \to G_0$ and $\varphi\colon H_1 \to G_1$, together commuting with all the structure maps in (1). Such a homomorphism is said to be an *essential equivalence* if (i), φ induces a surjective submersion $(y, g) \mapsto t(g)$ from the space $H_0 \times_{G_0} G_1 = \{(y, g) | \varphi(y) = s(g)\}$ onto H_0; and (ii), φ induces a diffeomorphism $h \mapsto (s(h), \varphi(h), t(h))$ from H_1 to the pullback $H_0 \times_{G_0} G_1 \times_{G_0} H_0$. Two Lie groupoids G and G' are said to be (*Morita*) *equivalent* if there are essential equivalences $G \leftarrow H \rightarrow G'$ from a third Lie groupoid H. (This notion is also often formulated in terms of principal bundles.)

A Lie groupoid G is said to be *étale* (or r-discrete) if all the structure maps in (1) are local diffeomorphisms (it is enough to require this for s). The relevance of étale groupoids for foliations is based on the following proposition [6].

Proposition 2.1. *A Lie groupoid is a foliation groupoid iff it is equivalent to an étale groupoid.*

If G is an étale groupoid, each arrow $g\colon x \rightarrow y$ in G uniquely determines the germ of a diffeomorphism $\tilde{g}\colon (G_0, x) \rightarrow (G_0, y)$, namely $\tilde{g} = \text{germ}_x(t \circ \sigma)$ where σ is a section of $s\colon G_1 \rightarrow G_0$ on a neighbourhood U of x, with $\sigma(x) = g$ and U so small that $t \circ \sigma$ is a diffeomorphism from U onto its image. This construction gives in particular a group homomorphism $G_x \rightarrow \text{Diff}_x(G_0)$. If this homomorphism is injective for each $x \in G_0$ the groupoid G is said to be *effective*. Effectivity is preserved under equivalence of groupoids, so we can define a foliation groupoid to be effective if it is Morita equivalent to an effective étale groupoid.

A groupoid G is called *proper* if $(s,t)\colon G_1 \rightarrow G_0 \times G_0$ is a proper map. An *orbifold groupoid* is a proper effective foliation groupoid. This notion is again invariant under equivalence. It can be shown that a Lie groupoid is an orbifold groupoid iff it is equivalent to the action groupoid associated to an infinitesimally free action of a compact Lie group on a manifold (see e.g. [15]).

3. The Holonomy Groupoid of a Foliation

Let $(M, \mathcal{F})$ be a foliated manifold. The holonomy groupoid $H = \text{Hol}(M, \mathcal{F})$ is a smooth groupoid with $H_0 = M$ as space of objects. If $x, y \in M$ are two points on different leaves there are no arrows from x to y in H. If x and y lie on the same leaf L, an arrow $h\colon x \rightarrow y$ in H (i.e. a point $h \in H_1$ with $s(h) = x$ and $t(h) = y$) is an equivalence class $h = [\alpha]$ of smooth paths $\alpha\colon [0,1] \rightarrow L$ with $\alpha(0) = x$ and $\alpha(1) = y$. To explain the equivalence relation, let T_x and T_y be small q-disks through x and y, transverse to the leaves of the foliation. If $x' \in T_x$ is a point sufficiently close to x on a leaf L' , then α can be "copied" inside L' to give a path α' near α with endpoint $y' \in T_y$, say. In this way one obtains the germ of a diffeomorphism from T_x to T_y, sending x to y and x' to y'. This germ is called the holonomy of α and denoted $\text{hol}(\alpha)$. By definition, two paths α and β from x to y in L are equivalent, i.e. define the same arrow $x \rightarrow y$ in H, iff $\text{hol}(\alpha) = \text{hol}(\beta)$. For example, if α and β are homotopic (inside L and relative endpoints) then $\text{hol}(\alpha) = \text{hol}(\beta)$. Composition and inversion of paths respects the equivalence relation, so that one obtains a well defined groupoid $H = \text{Hol}(M, \mathcal{F})$, which can be shown to be smooth [18]. This groupoid is a foliation groupoid, and the (discrete) isotropy group H_x at x is called the holonomy group of the leaf through x. If $T \subseteq M$ is an embedded q-manifold transverse to the leaves and hitting each leaf at least once, then the restriction of H to T defines an étale groupoid

$H_T = \mathrm{Hol}_T(M, \mathcal{F})$, equivalent to H. We refer to H_T as "the" étale (model for the) holonomy groupoid of $(M, \mathcal{F})$.

Morally, every étale groupoid G is the holonomy groupoid of a foliation (for a precise formulation, see [16, p. 21]). Orbifold groupoids are exactly the groupoids which arise as holonomy groupoids of foliations with compact leaves and finite holonomy groups [15].

The reader is urged to work out the various étale models for the holonomy groupoids of the Kronecker and Reeb foliations. He will notice that in the second case the space H_1 is a non-Hausdorff manifold.

The Haefliger groupoid Γ_q has $\mathbb{R}^q$ for its space of objects, while the arrows $x \to y$ are the germs of diffeomorphisms $(\mathbb{R}^q, x) \to (\mathbb{R}^q, y)$. When this space of arrows is equipped with the sheaf topology, Γ_q becomes an étale smooth groupoid. If $(M, \mathcal{F})$ is a foliation of codimension q, there is an essentially unique map of étale groupoids $\mathrm{Hol}_T(M, \mathcal{F}) \to \Gamma_q$ (for a suitable choice of T).

4. The Classifying Space

For a smooth groupoid G, the nerve of G is the simplicial set whose n-simplices are strings of composable arrows in G,

$$x_0 \xleftarrow{g_1} x_1 \longleftarrow \cdots \xleftarrow{g_n} x_n .$$

This set is denoted G_n, consistent with the earlier notation for $n = 0, 1, 2$. The space G_n is a fibered product $G_1 \times_{G_0} \cdots \times_{G_0} G_1$, hence has the natural structure of a smooth manifold. Thus $G_\bullet$ is a simplicial manifold. Its geometric realization is denoted BG and is called the classifying space of G. This construction respects Morita equivalence. In fact, an essential equivalence $H \to G$ induces a weak homotopy equivalence $BH \to BG$.

For a foliated manifold $(M, \mathcal{F})$ with holonomy groupoid H, there is a canonical 'quotient' map $M \to BH$, and BH models the space of leaves $M/\mathcal{F}$. One can also construct maps, canonical up to homotopy, $M \to BM_T \to B\Gamma_q$.

One of the problems with the space BH is that it is usually non-Hausdorff. There is another model for BH which doesn't have this defect. This model is based on a small (discrete) category $\mathrm{Emb}(G)$ constructed for any étale groupoid G. (In this context, $G = H_T$ is the étale model for the holonomy groupoid.) The objects of this new category $\mathrm{Emb}(G)$ are the members of a fixed basis of contractible open sets for the topology on G_0. For two such basic open U and V, each section $\sigma\colon U \to G_1$ of the source map, with the property that $t \circ \sigma\colon U \to G_0$ defines an embedding into V, defines an arrow $\hat{\sigma}\colon U \to V$ in the category $\mathrm{Emb}(G)$. Composition is defined by $\hat{\tau} \circ \hat{\sigma} = \hat{\rho}$ where $\rho(x) = \tau(t\sigma(x))\sigma(x)$ (multiplication in G). The nerve of this category $\mathrm{Emb}(G)$ is a simplicial set, whose geometric realization is denoted $B\,\mathrm{Emb}(G)$.

Theorem 4.1. [12] *For any étale groupoid G the spaces BG and $B\,\mathrm{Emb}(G)$ are weakly homotopy equivalent.*

For the special case where $G = \Gamma_q$, the category $\mathrm{Emb}(G)$ is categorically equivalent to the (discrete) monoid M_q of smooth embedding of $\mathbb{R}^q$ into itself, and one recovers Segal's theorem, $B\Gamma_q \simeq BM_q$.

We remark that, unlike BG, the classifying space $B\mathrm{Emb}(G)$ is a CW-complex, hence within the scope of the usual methods of algebraic topology. It is also very well suited for the explicit geometric construction of characteristic classes of foliations [6].

5. The Classifying Topos

Let G be an étale (or foliation) groupoid. A G-sheaf of sets is a sheaf on G_0 equipped with a continuous right action by G. When $\pi\colon S \to G_0$ is the étale space of the sheaf, the action can be described as a continuous map $S \times_{G_0} G_1 \to S$. For $\xi \in S_y = \pi^{-1}(y)$ and $g\colon x \to y$, the result of the action is denoted $\xi \cdot g \in S_x$. The usual identities $\xi \cdot 1_y = \xi$ and $(\xi \cdot g) \cdot h = \xi \cdot (gh)$ are required to hold. With the obvious notion of action preserving map, these sheaves form a category $\mathrm{Sh}(G)$. This category is a topos [17], called the classifying topos of G, and discussed in [13]. A homomorphism of groupoids $G \to H$ induces a topos map $\mathrm{Sh}(G) \to \mathrm{Sh}(H)$. The construction preserves Morita equivalence. (In fact, $\mathrm{Sh}(G)$ and $\mathrm{Sh}(H)$ are equivalent toposes, i.e. are equivalent as categories, if *and only if* G and H are Morita equivalent as topological —rather than smooth— groupoids.)

If follows that the category $\mathrm{Ab}\,\mathrm{Sh}(G)$ of sheaves of abelian groups has enough injectives, and one obtains for each sheaf A the sheaf cohomology groups $H^n(G, A)$ as those of the topos, i.e. $H^n(G, A) = H^n(\mathrm{Sh}(G), A)$ for $n \geq 0$ by definition. These cohomology groups are then automatically contravariant in G and invariant under Morita equivalence, and satisfy all the usual general properties of [17] (Leray spectral sequence, Čech spectral sequence, hypercover description, relation of $H^1(G, A)$ to the fundamental group of $\mathrm{Sh}(G)$, etc. etc.).

This approach is compatible with the classifying space, as follows.

Theorem 5.1. *Any abelian G-sheaf A induces in a natural way a sheaf $\tilde{A}$ on the classifying space BG, and one has a canonical isomorphism*

$$H^n(G, A) \xrightarrow{\sim} H^n(BG, \tilde{A}), \quad n \geq 0.$$

This isomorphism was conjectured by Haefliger and proved in [14].

There is also a dual *homology theory* for étale groupoids, introduced and studied in [7]. For an abelian G-sheaf A, we construct homology groups

$$H_n(G, A), \quad n \geq -\dim(G),$$

again invariant under Morita equivalence of groupoids and having good general properties. In particular, there is a Verdier type duality between the cohomology just described and this homology.

It seems difficult to give a description of this homology theory in terms of the classifying space BG, although in some special cases this is possible.

6. The Convolution Algebra and Cyclic Homology

For an algebra A, one can define the Hochschild, cyclic and periodic cyclic homology groups, denoted $HH_n(A), HC_n(A)$ ($n \in \mathbb{N}$) and $HP_\nu(A)$ ($\nu = 0, 1$). The definition is based on iterated tensor products $A \otimes \cdots \otimes A$. In the special case where $A = C_c^\infty(M)$ is the ring of smooth compactly supported functions on a manifold, a well-known result of Connes' [5], which played a central role in the development of cyclic homology, provides the following relation to the De Rham cohomology,

$$HH_n(C_c^\infty(M)) = \Omega_c^n(M)\,, \quad HP_\nu(M) = H_c^\nu(M)\,. \tag{2}$$

Here $\Omega_c^n(M)$ is the vector space of compactly supported n-forms on M, and $H_c^\nu(M)$ is the product of the even ($\nu = 0$) or odd ($\nu = 1$) compactly supported De Rham cohomology groups.

It is important to note that for this result, the algebraic tensor product $A \otimes B$ is replaced by a completed topological tensor product (the inductive one) $A \hat{\otimes} B$ having the property that $C_c^\infty(M) \hat{\otimes} C_c^\infty(N) = C_c^\infty(M \times N)$ for two manifolds M and N.

Connes' result extends to generalized manifolds such as leaf spaces of foliations. More specifically, let G be an étale groupoid. The convolution algebra $C_c^\infty(G)$ is the algebra of compactly supported smooth functions a, b, $\ldots$ on G_1, with "convolution" product

$$(a * b)(g) = \sum_{g=hk} a(h) b(k)\,,$$

exactly as for the group ring. (The sum here makes sense, because it ranges over a space which is discrete because G is étale and finite because of compact supports.) Using the inductive topological tensor product $\hat{\otimes}$, one can then define the "cyclic type" homology groups $HH_*(C_c^\infty(G))$, $HC_*(C_c^\infty(G))$ and $HP_*(C_c^\infty(G))$. The construction of the convolution algebra $C_c^\infty(G)$ is not functorial in G, and the invariance under Morita equivalence of these cyclic type homology groups is established in a rather indirect way, by relating them to the homology groups mentioned above, as follows.

For an étale groupoid G, the 'loop groupoid' $\Lambda(G)$ has as its objects the arrows $g \colon x \to x'$ in G with $x = x'$. Arrows $g \to h$ in $\Lambda(G)$, from $(g \colon x \to x)$ to $(h \colon y \to y)$, are arrows $\alpha \colon x \to y$ in G with $h\alpha = \alpha g$. This groupoid $\Lambda(G)$ is a topological étale groupoid. This construction is functorial in G, and preserves Morita equivalence. There is also an evident retraction $\pi \colon \Lambda(G) \to G$.

Let $\mathcal{A}_n$ be the pullback along the diagonal $G_0 \to G_0^{n+1}$ of the sheaf of smooth functions on G_0^{n+1}. The stalk of $\mathcal{A}_n$ at x is the ring $C_x^\infty(G_0) \hat{\otimes} \cdots \hat{\otimes} C_x^\infty(G_0)$ and consists of germs of functions $f(x_0, \ldots, x_n)$. The usual Hochschild boundary of the complex $\mathcal{A}_0 \xleftarrow{b} \mathcal{A}_1 \leftarrow \cdots$ can be twisted by loops to give a complex $\pi^*(\mathcal{A}_0) \xleftarrow{b} \pi^*(\mathcal{A}_1) \leftarrow \cdots$ on $\Lambda(G)$, and one has the following comparison,

$$HH_*(C_c^\infty(G)) = H_*(\Lambda(G), \pi^*(\mathcal{A}))\,, \tag{3}$$

expressing the Hochschild homology of the convolution algebra in terms of the (hyper-)homology of the étale groupoid $\Lambda(G)$. An immediate consequence is that $HH(C_c^\infty(G))$ is invariant under Morita equivalence of étale groupoids. (The invariance of HC and HP follows by the usual SBI-argument.) If G is a manifold, i.e. $G_0 = G_1 = M$, then $\Lambda(G) = G = M$ also, and one recovers Connes' isomorphism (2) from (3). If $G = \Gamma$ is a discrete group, $\Lambda(\Gamma)$ is Morita equivalent to the sum over all conjugacy classes of centralizer subgroups,

$$\Lambda(\Gamma) \cong \sum_{(\gamma)} Z_\gamma ,$$

and one recovers the well-known description of the Hochschild homology of the group ring ([3, 10]). If $G = M \rtimes \Gamma$ is the action groupoid associated to the action of a discrete group Γ on a manifold M, then $\Lambda(G)$ is Morita equivalent to $\bigoplus_{(\gamma)} M^\gamma \rtimes Z_\gamma$, giving a familiar decomposition of $HH_*(C_c^\infty(M \rtimes \Gamma))$.

There are isomorphisms similar to (3) for the cyclic and periodic cyclic homology groups of étale groupoids, all relating these groups to the homology of étale groupoids (or categories) like $\Lambda(G)$. For precise formulations and computations I refer to [1, 8, 7]. Here as an illustration, I just single out the special case where G is an étale groupoid with finite isotropy groups (e.g. an orbifold groupoid). In this case, one has a natural isomorphism

$$HP_\nu(C_c^\infty(G)) = \prod_k H_{2k+\nu}(\Lambda(G), \mathbb{R}), \quad (\nu = 0, 1) ,$$

extending Connes' isomorphism (2) to such groupoids.

References

[1] J.-L. Brylinski, V. Nistor, *Cyclic cohomology of étale groupoids*, K-theory **8** (1994), 341–365.

[2] R. Bott, *Lectures on characteristic classes and foliations*, Springer LNM **279**, 1–94.

[3] D. Burghelea, *The cyclic cohomology of group rings*, Comm. Math. Helv. **60** (1985), 354–365.

[4] A. Connes, *Noncommutative Geometry*, Academic Press, 1994.

[5] A. Connes, *Noncommutative differential geometry*, Publ. IHES **62** (1985), 41–144.

[6] M. Crainic, I. Moerdijk, *Cech-De Rham theory for leaf spaces of foliations*, in prep.

[7] M. Crainic, I. Moerdijk, *A homology theory for étale groupoids*, to appear in Crelle's J. f. reine und angewandte Mathematik.

[8] M. Crainic, *Cyclic cohomology of étale groupoids: The general case*, K-Theory **17** (1999), 319–362.

[9] A. Haefliger, *Feuilletages sur les variétés ouvertes*, Topology **9** (1970), 183–194.

[10] M. Karoubi, *Homologie cyclique des groupes et des algèbres*, CRAS **297** (1983), 381–384.

[11] K. Mackenzie, *Lie Groupoids and Lie Algebroids in Differential Geometry*, Cambridge U.P., 1987.

[12] I. Moerdijk, *On the weak homotopy type of étale groupoids*, in "Integrable Systems and Foliations", Birkhäuser (1997), 147–156.

[13] I. Moerdijk, *Classifying toposes and foliations*, Ann. Inst. Fourier **41** (1991),189–209.

[14] I. Moerdijk, *Proof of a Conjecture of A. Haefliger*, Topology **37** (1998), 735–741.

[15] I. Moerdijk, D. Pronk, *Orbifolds, Sheaves and Groupoids*, K-Theory **12** (1997), 3–21.

[16] J. Mrcun, *Stability and invariance of Hilsum-Skandaliz maps*, PhD Thesis, Utrecht, 1996.

[17] M. Artin, A. Grothendieck, J.-L. Verdier, *Théorie de topos et cohomologie étale des schémas ("SGA IV")*, Springer Lecture Notes 269, 270 (1972).

[18] H. Winkelnkemper, *The graph of a foliation*, Ann. Global Anal. Geom. **1** (1983), 613–665.

Mathematical Institute
University of Utrecht
PO Box 80.010
3508 TA Utrecht, The Netherlands
E-mail address: moerdijk@math.uu.nl