Inversion of $\delta(^{13}C)$ of CO_2 with TM5MP and CTDAS

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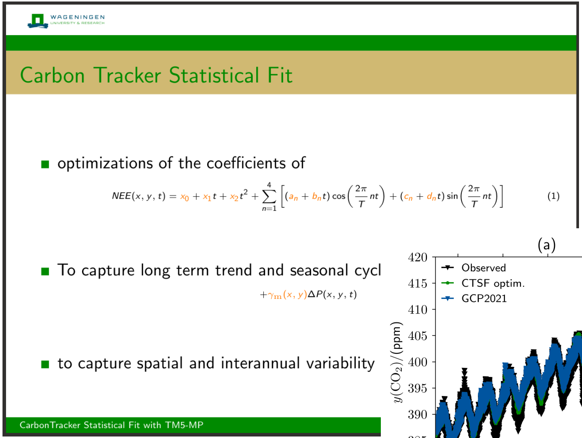
TM5 meeting, 19–20 December 2022

Recap: Dual tracer inversion with $\delta(^{13}C)$ and CO_2 observations

Inversions using CO_2 mole fractions and $\delta(^{13}C)$ to learn more about carbon uptake of the ocean and biosphere

Last TM5-meeting



Carbon Tracker Statistical Fit

- optimizations of the coefficients of

$$NEE(x,y,t) = x_0 + x_1 t + x_2 t^2 + \sum_{n=1}^{4}\left[(a_n + b_n t)\cos\left(\frac{2\pi}{T}nt\right) + (c_n + d_n t)\sin\left(\frac{2\pi}{T}nt\right)\right] \quad (1)$$

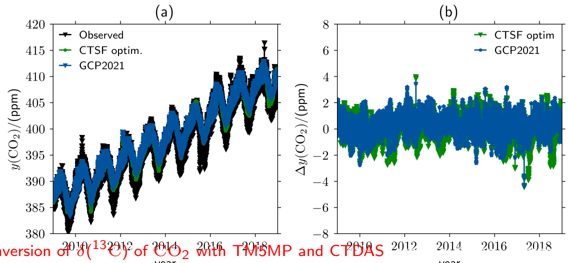
- To capture long term trend and seasonal cycl

$$+\gamma_{\mathrm{m}}(x,y)\Delta P(x,y,t)$$

- to capture spatial and interannual variability

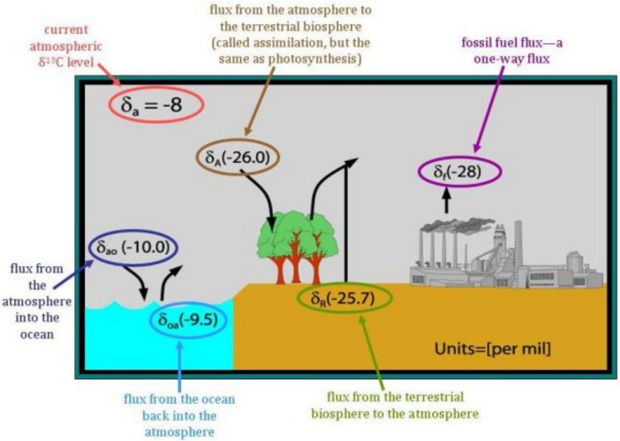
CarbonTracker Statistical Fit with TM5-MP

Wouter's talk



Carbon-13 budget

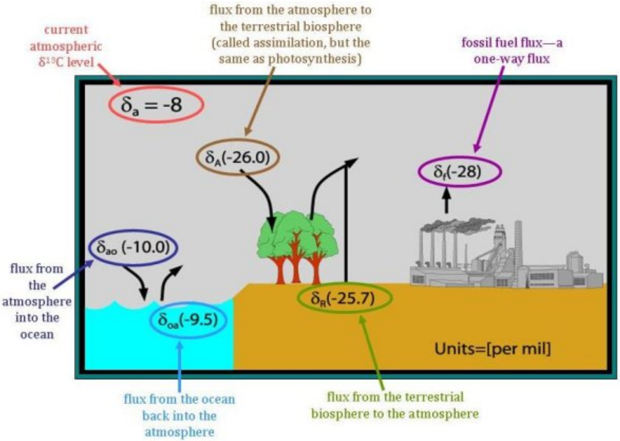
https://gml.noaa.gov/ccgg/isotopes/c13tellsus.html



Fractionation changes the relative ^{13}C content

Carbon-13 budget

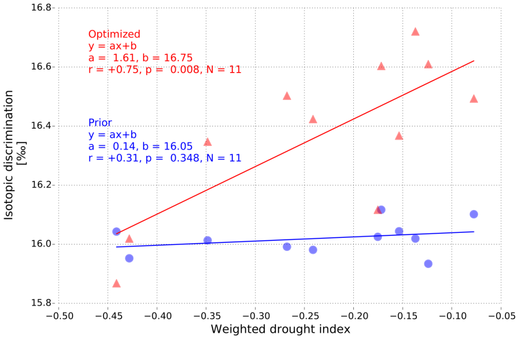
https://gml.noaa.gov/ccgg/isotopes/c13tellsus.html

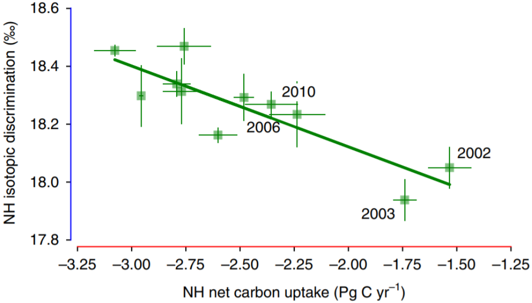


Fractionation changes the relative ^{13}C content Droughts impact the overall fractionation

Why carbon-13 in a CO_2 inversion

(Velde et al., 2018)





(Peters et al., 2018)

Developments in 13-Carbon

1. Development in a multi-tracer inversion framework
   - CTDAS and TM5-MP

TM5-MP & CTDAS

Multi-tracer framework focussing on the carbon cycle:

CO_2, $\delta(^{13}CO_2)$, and CO

1. TM5-MP git branch: `wur/ctecc`
2. CTDAS git branch: `ctecc`

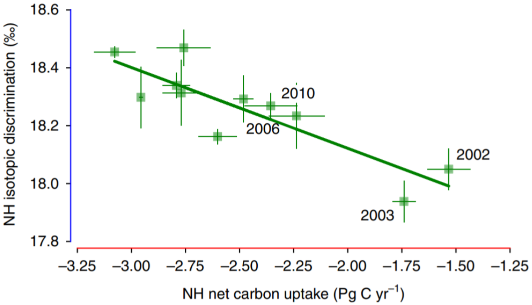
1. Obspack flask sampling
   1. `proj/enkf/base/user_output_flask.F90`
   2. `da/observations/obs_ctecc.py`
2. OCO2 column sampling
   1. `proj/enkf/base/user_output_column.F90`
   2. `da/observations/obs_column.py`

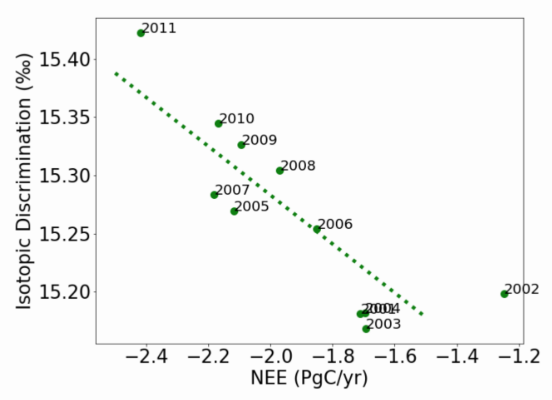
Developments in 13-Carbon

1. Development in a multi-tracer inversion framework
2. Improved carbon-13 budget
   - Reanalysis of $\delta(^{13}C)$ in DIC for ocean isofluxes
   - More $\delta(^{13}C)$ observations: space and time
   - Sophisticated $\delta(^{13}C)$ scheme in SiB4

Improved carbon-13 budget

SiB4 by Aleya Kaushik (NOAA)

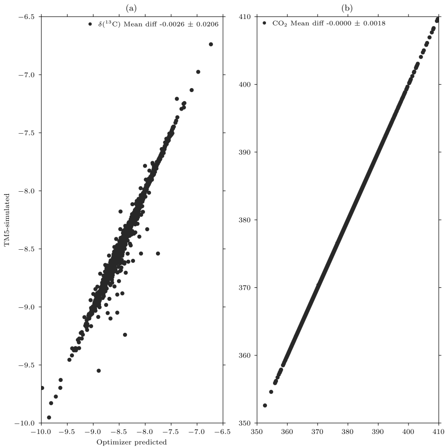




Developments in 13-Carbon

1. Development in a multi-tracer inversion framework
2. Improved carbon-13 budget
3. Assimilate $^{13}CO_2$ and CO_2 mole fractions with CTE long-window to give posterior
   - biosphere fluxes
   - ocean fluxes
   - fractionation
   - disequilibrium fluxes (isotope only)
4. One-shot inversion: the no-TM5-inversion

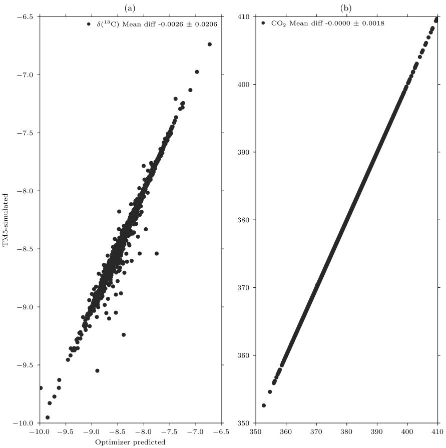
TM5 is a linear operator for CO_2



x-axis: optimizer posterior
y-axis: TM5 re-run with posterior fluxes

δ-values have a dependence on 1/TM5 and products of statevector elements

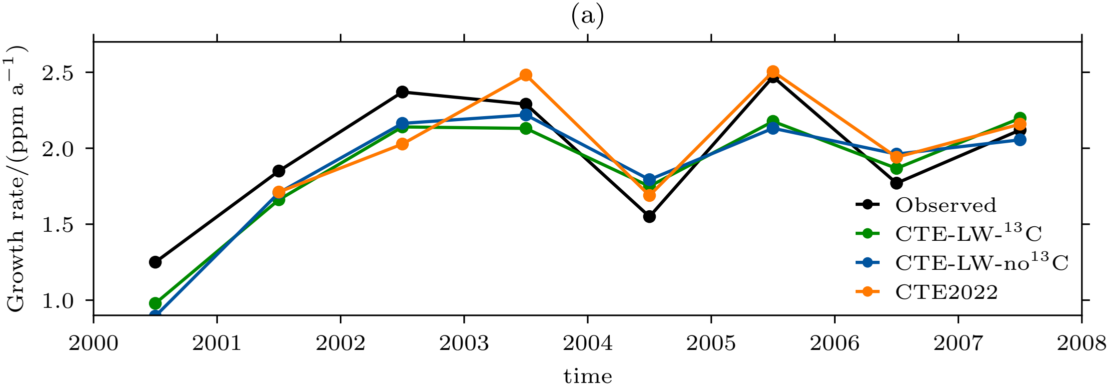
TM5 is a linear operator for CO_2



x-axis: optimizer posterior
y-axis: TM5 re-run with posterior fluxes

δ-values have a dependence on 1/TM5 and products of statevector elements
no-$\delta(^{13}C)$ inversion for free

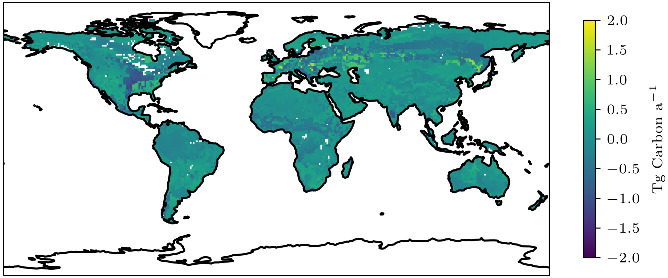
Results: global growth rate of atmospheric CO_2



$\delta(^{13}C)$ vs no $\delta(^{13}C)$

Identical answers:

- Mean biospheric uptake 4.4 PgC a^{-1}
- Mean ocean uptake 1.7 PgC a^{-1}
- But different spatial patterns:



Developments in 13-Carbon

1. Development in a multi-tracer inversion framework
2. Improved carbon-13 budget
3. Assimilate $^{13}CO_2$ and CO_2 mole fractions with CTE long-window to give posterior
4. One-shot inversion: the no-TM5-inversion
   - number of parameters $N = 636888$
   - More observations (Satellite?)

Ensemble Kalman Filter in Fortran with a Python wrapper

```fortran
! calculate PHt using blas/lapac DGEMV
! Computes PHt = 1/(N-1) HXp_n . Xp
call DGEMV('N',nparams,nmembers,n_fac,Xp,nparams,HXp_n,1,0.0,PHt,1)

! estimates HPH+R=1/(N-1) HXp_n . HXp_n + R   , using DDOT.
HPHR(i) = n_fac * DDOT(nmembers,HXp_n,1,HXp_n,1) + R(i)

! Compute the Kalman Gain
KG = PHt/HPHR(i)

! alpha factor
alpha=1.0 / (1.0 + sqrt( R(i) / HPHR(i) ))

! We now have the info to compute the updated version of
! x, Xp, HXp, and Hx
! some of this stuf is not final yet

! could it be more effiecient to do this and the part of fac in order to update Hx?
! call DGEMV('N',nobs,nmembers,res*n_fac*(1/HPHR(i)),HXp,nmembers,HXp(i,:),1,1.0,Hx,1)

! Approximate HK, and store in fac
call DGEMV( 'N' , nobs , nmembers , n_fac * (1.0 / HPHR(i)),HXp,nobs,HXp_n,1,0.0,fac,1)

! update Hx = Hx + HK * residual
call daxpy(nobs,res,fac,1,Hx,1)

! update x = x  + res * KG
call daxpy(nparams,res,KG,1,x,1)

! update deviations HX = HX - HK HXp(i) using outer product
! call DGER(nobs,nmembers,-1.0*alpha,fac,1,HXp_n,1,HXp,nobs)
```

1. Fortran source on Github
2. Python wrapper F2PY
3. $>$ 2 times faster
4. Use with ctdas: `da/optimizers/optimizer_ctecc.py`

```python
pyenkf.enkf_core.enksrf(self.obs,
                        self.may_reject,
                        self.rejection_threshold,
                        self.Hx,
                        self.HX_prime,
                        self.X_prime,
                        self.R,
                        self.HPHR,
                        self.x,
                        rejected)
```

Developments in 13-Carbon

1. Development in a multi-tracer inversion framework
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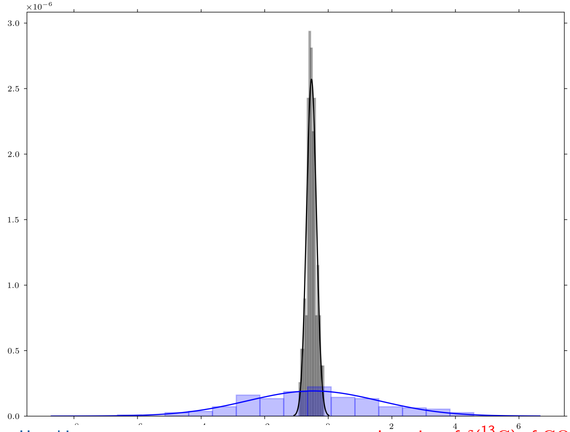
4. One-shot inversion: the no-TM5-inversion
   - number of parameters $N = 636888$
   - More observations (Satellite?)
5. Analysis of covariance structure

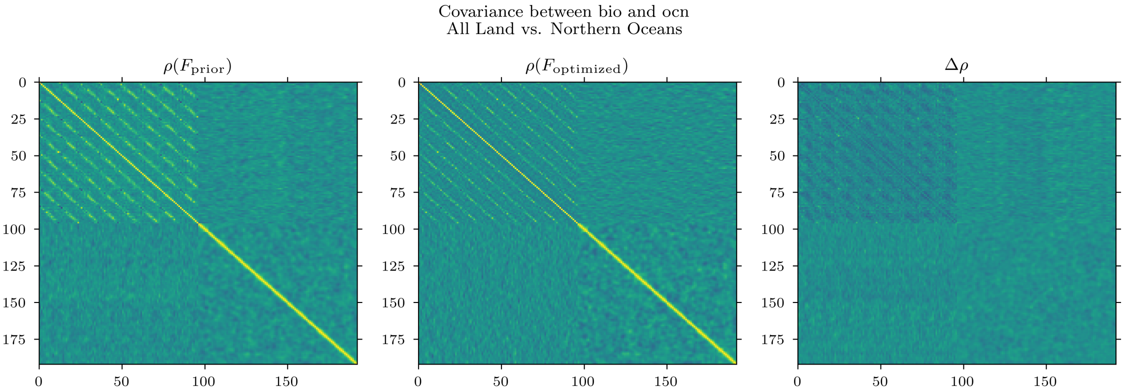
- Ensemble with 150 members:
  - 150 different emission fields prior and posterior



- Ensemble with 150 members:
  - 150 different emission fields
  - 150*number of species of tracers in TM5
  - 150 different versions of an observations

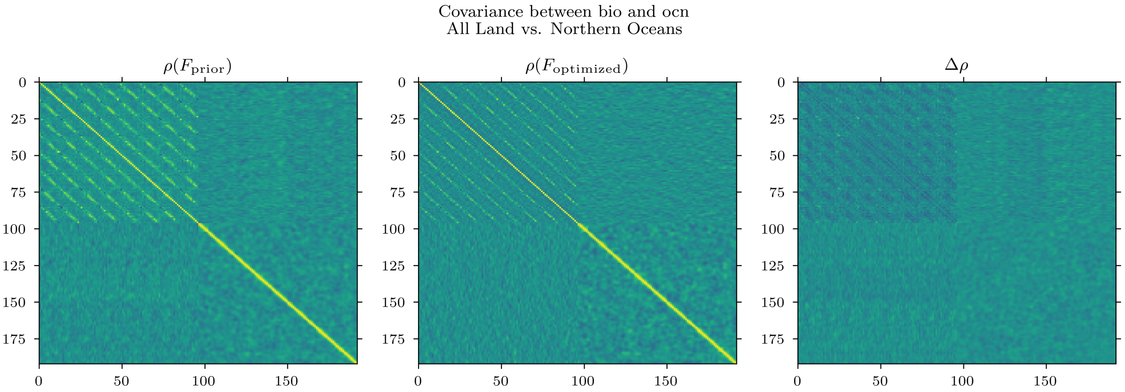
Analysis of covariance

x and y-axis contain 2 time series: month 1 to 96



Analysis of covariance

Each pixel is the correlation between flux F_1 at month x_1 with flux F_2 at x_2

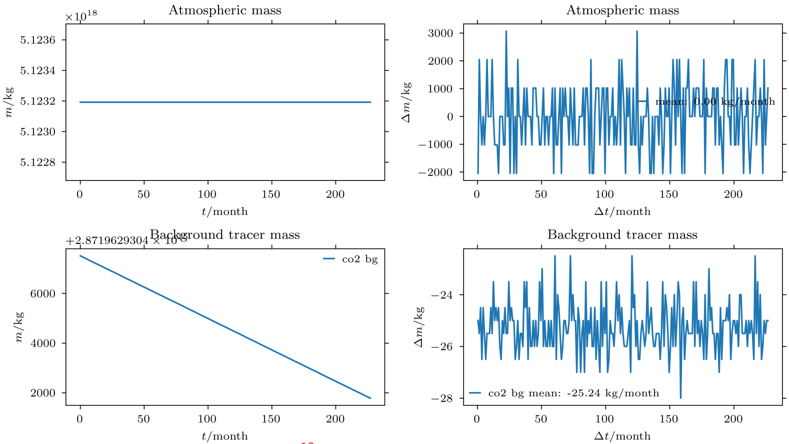


Summary

1. Development in a multi-tracer inversion framework
2. Improved carbon-13 budget
3. Assimilate $^{13}CO_2$ and CO_2 mole fractions with CTE long-window to give posterior

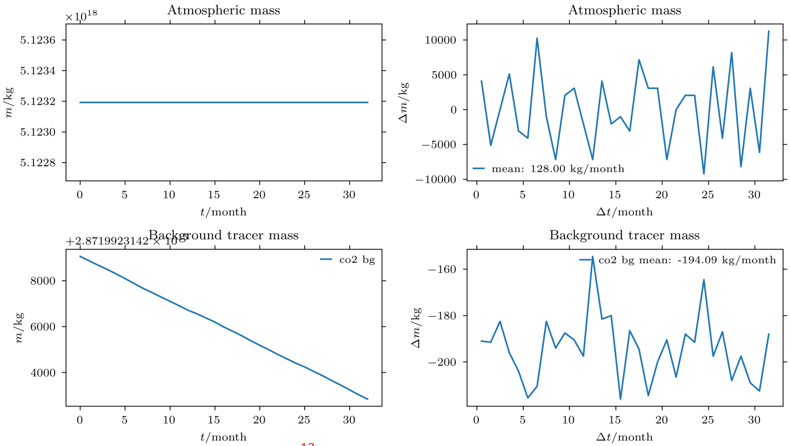
Mass conservation

TM5-MP 6x4



Mass conservation

TM5-MP 1x1



Mass conservation

TM5-Zoom